MATH 3D Prep: Integration by Parts

1. Evaluate $\int 2(t+2)\cos(4t)dt$

Solution: Choose $u = 2(t+2)$, $dv = \cos(4t)dt$, then $du = 2dt$, $v = \frac{1}{4}\sin(4t)$, so

$$\begin{aligned}\int 2(t+2)\cos(4t)dt &= \int u\,dv = uv - \int v\,du \\ &= \frac{(t+2)\sin(4t)}{2} - \int \frac{1}{4}\sin(4t)\cdot 2dt \\ &= \frac{(t+2)\sin(4t)}{2} - \int \frac{1}{2}\sin(4t)dt \\ &= \frac{(t+2)\sin(4t)}{2} + \frac{1}{8}\cos(4t) + C.\end{aligned}$$

2. Evaluate $\int e^{2x}\sin(x)dx$

Solution: Taking $u = e^{2x}$, $dv = \sin(x)dx$, then $du = 2e^{2x}dx$, $v = -\cos(x)$, so

$$\int e^{2x}\sin(x)dx = -e^{2x}\cos(x) - \int(-\cos x)\cdot 2e^{2x}dx = -e^{2x}\cos(x) + 2\int e^{2x}\cos(x)dx \qquad (1)$$

Similarly, Take $u = e^{2x}$, $dv = \cos(x)dx$, then $du = 2e^{2x}dx$, $v = \sin(x)$, compute

$$\int e^{2x}\cos(x)dx = e^{2x}\sin(x) - 2\int e^{2x}\sin(x)dx \qquad (2)$$

Plugging equation (2) into equation (1), get

$$\int e^{2x}\sin(x)dx = -e^{2x}\cos(x) + 2\left(e^{2x}\sin(x) - 2\int e^{2x}\sin(x)dx\right).$$

Solve for $\int e^{2x}\sin(x)dx$ and remembering the constant C for indefinite integrals, get

$$\int e^{2x}\sin(x)dx = \frac{1}{5}\left(-e^{2x}\cos(x) + 2e^{2x}\sin(x)\right)$$